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| Mathematics Design Collaborative |
| State of Georgia Department of Education |



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| **Linear and Exponential Functions** |
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| ***Exploring Paths*** |
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| **INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON** |
| **MATHEMATICAL GOALS** |
| This lesson unit is intended to help you assess how well students are able to: |
| * Utilize what they already know about linear functions and exponential functions in the context of different graphs. * Reasoning qualitatively, compares linear and exponential models verbally, numerically, algebraically, and graphically. |

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| **GEORGIA STANDARDS OF EXCELLENCE** |
| This lesson involves mathematical content in the standards from across the grades, with emphasis on: |
| **MGSE9-12.F.BF.1** Write a function that describes a relationship between two quantities.  **MGSE9-12.F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.  **MGSE9-12.F.LE.1a** **Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).**  **MGSE9-12.F.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.  **MGSE9-12.F.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| This lesson involves a range of mathematical practices, with emphasis on:   * SMP4. Model with mathematics. * SMP7. Look for and make use of structure. * SMP8. Look for and express regularity in repeated reasoning. |
| **INTRODUCTION** |
| This lesson is structured in the following way: |
| Before the Lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions. |
| At the Start of the Lesson, a whole class discussion should include real-world situations resulting in both a linear model and an exponential model. See teacher guide pages. |
| During the Lesson, students work in pairs on a collaborative discussion task in which:   * Students first match verbal situation (Card Set A) with the modeling function (Card Set B). * Students will then match a possible graph (Card Set C) that could go with the verbal situation and modeling function. |
| After the Whole-Group Class Discussion, students return to their original assessment tasks, and try to improve their own responses. |
| **MATERIALS REQUIRED** |
| Each individual student will need a copy of the Pre-Assessment and a copy of the Post-Assessment |
| Each small group of students will need:  a set of card set A, a set of card set B, a set of card set C, Poster Paper, Glue Sticks or Tape |

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| **TEACHER PREP REQUIRED** |
| Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made: |
| * Copies of the pre and post-assessment for each student * Group sets of card sets A, B, C (copy and cut apart) * Poster paper , glue or tape * Projector resource slide of linear and exponential graph |

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| For Pre-Assessment: 15 minutes | For Lesson: 45 minutes | For Post: 15 minutes |

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| **FRAMING FOR THE TEACHER:** |
| CCGPS Mathematics compares exponential to linear functions during 9th grade in order for students to understand the difference between a constant rate of change and a non-constant rate of change. These two are chosen for comparison because of the arithmetic vs. geometric link to sequences.  Students learn that a constant difference creates an arithmetic sequence when the domain is comprised of integer values, whereas a constant factor creates a geometric sequence under the same circumstances. This task highlights the differences between the two types of functions and their rates of change. The collaborative activity requires that student understand how to create equations to model situations that may be described by one of these types of functions. |
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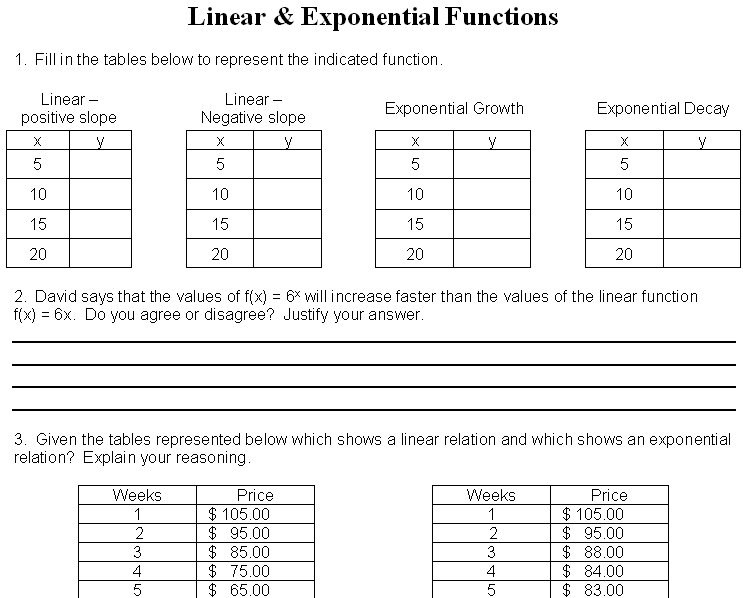
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| **FRAMING FOR THE KIDS:** |
| Say to the students: |
| *This activity will take about 2 days for us to complete.* |
| *The reason we are doing this is to be sure that you understand the difference between linear functions with a constant rate of change and exponential functions with a non-constant rate of change before we move on to a new idea.* |
| *You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!* |

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| **PRE-ASSESSMENT BEFORE THE LESSON** |
| **ASSESSMENT TASK:** |
| Name of Assessment Task: *Understanding Paths* |
| Time This Should Take: 15 minutes |

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will them be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the Pre-Assessment:

Briefly introduce the task and help the class to understand the problem and its context.

*Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don’t worry if you can’t complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.*

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them with *Knowing how to read a graph from left to right.* Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.

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| **COLLABORATION TIME/READING STUDENTS RESPONSES** |
| **You Will Not “Grade” These!** |
| Collect students’ responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another, and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: pairs are better than larger groups for FAL’s because both must participate in order to discuss!) |
| You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions, however some guiding questions and prompts are also listed below as a jumping-off point. |

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| **GUIDING QUESTIONS** | |
| ***COMMON ISSUES*** | ***SUGGESTED QUESTIONS AND PROMPTS*** |
| **Knowing how to read a graph from left to right.** | * *How do you read a graph?* |
| **Understanding slope** | * *What is slope?* * *How can you find the slope?* * *How can you find slope using two points?* * *What does it say about a graph when the slope decreases? Increases? Is constant?* |
| **Comparing a number raised to a power versus a number times a number.**  Ex. 63 and 6(3) | * *What does it mean to raise a number to a power?* * *Compare a number to a power with a number times a number.* |

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| **LESSON DAY** | |
| **SUGGESTED LESSON OUTLINE:** | |
| **Part 1: Whole-Class Introduction:** | **Time to Allot: ( 10 minutes)** |
| Display the “Warm Up” question provided. | |
| **Whole-class interactive introduction (10 minutes)**  Discuss linear and exponential functions. There is a slide in Teacher Resources that the teacher may display during this discussion.  Linear Real-world Situation: Hourly Wage  **Possible Equation: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**  Exponential Real-World Situation: Population increasing over time.  http://image.tutorvista.com/cms/images/38/exponential-growth-graph.JPG  **Possible Equation: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** | |

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| **Part 2: Collaborative Activity:** | | **Time to Allot: ( 20 minutes)** | |
| Put students into their pairs according to your analysis of student errors.   * Group students in pairs by common errors found in Formative Assessment. * Students are given card sets A and B, already cut apart. * Students should match one card from A with one card from B. * After 10-15 minutes, give students card set C to match with previous sets. There are only 4 cards in card set C and so each card will match multiple cards from Sets A and B. * Using construction paper glue the matching cards from sets A & B underneath the appropriate graph from set C. | | | |
| Do/Say the Following: | | | |
| **Make a note of student approaches to the task.**  Listen and watch students carefully. Note different student approaches to the task and any common mistakes. For example, students may   |  | | --- | | * Group all the positive slopes together and all the negative slopes together. | | * Match cards that have the same numbers. | | * Compare graphs with constant slopes and those with rapidly changing slopes. |   **Support student reasoning**  Try not to make suggestions that steer students towards a particular “correct” answer or response. Instead, ask questions to help students to reason together.  If you find one student has produced a correct response, challenge another student in the group to provide an explanation.  *Example:*   * *Why do you think this is an exponential growth?* * *Why do you think this is an exponential decay?* * *What words in this problem made you think it is a linear function?* * *What words in this problem made you think it is an exponential function?*   If you find students have difficulty articulating their decisions, use the sheet Suggested Questions and Prompts to support your own questioning of students.  *Sample ways to jump-start students’ work in the group collaboration:*   |  | | --- | | * *How do you read a graph?* | | * *What is slope?* * *How can you find the slope?* | | * *What does it mean to raise a number to a power?* * *Compare a number to a power with a number times a number.* |   If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could ask students who performed well in the assessment to help struggling students.  Allow students time to collaborate as much as possible. | | | |
| During the Collaborative Activity, the Teacher has 3 tasks:   * Circulate to students’ whose errors you noted from the pre-assessment and support their reasoning with your guiding questions. * Circulate to other students also to support their reason in the same way. * Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions! | | | |
| **Part 3: Plenary (Summary) Discussion:** | | **Time to Allot: ( 15 minutes)** | |
| Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below: | | | |
| |  | | --- | | * *Students will share their card matching activity..* | | * *Why did you choose the given graph for the given situation?* * *How can the equation of a function help us to determine the shape of the graph?* * *In a given equation, what effect does the “m” have on the graph? The “n”?* |   NOTE: *“Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!* | | | |
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| **Part 4: Improving Solutions to the Assessment Task** | **Time to Allot: ( 20 minutes)** | |
| The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.  Practice will make perfect, and teachers should do what makes them most comfortable with their students/finds misconceptions!  *Look at your original responses and think about what you have learned this lesson.*  *Using what you have learned, try to improve your work.*  If you have not added questions to individual pieces of work then write your list of questions on the board.  Students should select from this list only the questions they think are appropriate to their own work.  If you find you are running out of time, then you could set this task in the next lesson or for homework. | | |

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| **ASSESSMENT TASK: Key** |
| **ANSWERS WILL VARY** |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Linear – positive slope | |  | Linear – Negative slope | |  | Exponential Growth | |  | Exponential Decay | | | x | y |  | x | y |  | x | y |  | x | y | | 5 | 5 |  | 5 | -11 |  | 5 | 32 |  | 5 | 1.9 | | 10 | 15 |  | 10 | -16 |  | 10 | 1,024 |  | 10 | .6 | | 15 | 25 |  | 15 | -21 |  | 15 | 32,768 |  | 15 | .2 | | 20 | 35 |  | 20 | -26 |  | 20 | 1,048,576 |  | 20 | .07 |   2. David says that the values of f(x) = 6x will increase faster than the values of the linear function f(x) = 6x. Do you agree or disagree? Justify your answer.  *Agree, grows exponentially and is a linear function growing at a constant rate.*  3. Given the tables represented below which shows a linear relation and which shows an exponential relation?   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Weeks | Price |  | Weeks | Price | | 1 | $ 105 |  | 1 | $ 105 | | 2 | $ 95 |  | 2 | $ 95 | | 3 | $ 85 |  | 3 | $ 86 | | 4 | $ 75 |  | 4 | $ 78 | | 5 | $ 65 |  | 5 | $ 71 |   Explain your reasoning.  *The first table is linear because it shows a constant rate of -10 and the second is exponential and shows a non-constant rate of change with a constant ratio of 0.9.*  4. Write a real-life situation that could be represented by the function f(t) = 3000(1.07)t . Explain.  *Answers will vary : Deposit $3000 in a bank that pays 7% yearly rate.* |

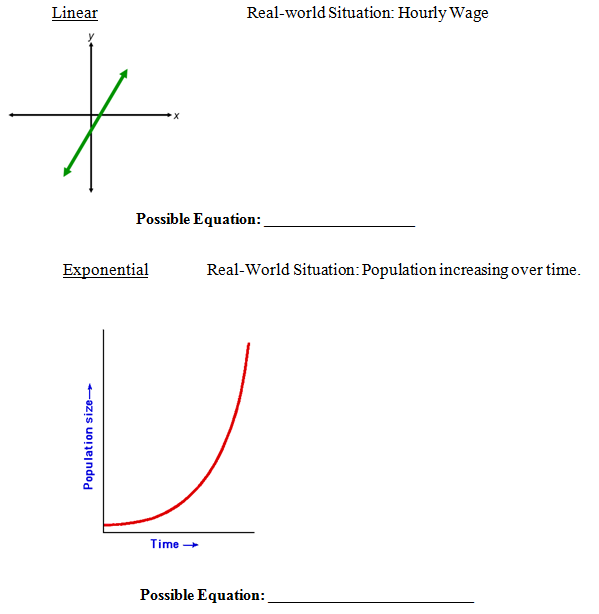
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| **Collaborative Activity (Answer Key)** |
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| |  |  |  | | --- | --- | --- | | A1 | B3 | C4 | | A2 | B5 | C4 | | A3 | B4 | C1 | | A4 | B6 | C1 | | A5 | B1 | C2 | | A6 | B2 | C3 | | A7 | B8 | C3 | | A8 | B7 | C2 | | A9 | B10 | C1 | | A10 | B9 | C4 | |

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| **Linear & Exponential Functions**  1. Fill in the tables below to represent the indicated function.   |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Linear – positive slope | |  | Linear – Negative slope | |  | Exponential Growth | |  | Exponential Decay | | | x | y |  | x | y |  | x | y |  | x | y | | 5 |  |  | 5 |  |  | 5 |  |  | 5 |  | | 10 |  |  | 10 |  |  | 10 |  |  | 10 |  | | 15 |  |  | 15 |  |  | 15 |  |  | 15 |  | | 20 |  |  | 20 |  |  | 20 |  |  | 20 |  |   2. David says that the values of f(x) = 6x will increase faster than the values of the linear function f(x) = 6x. Do you agree or disagree? Justify your answer.  3. Given the tables represented below which shows a linear relation and which shows an exponential relation?   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Weeks | Price |  | Weeks | Price | | 1 | $ 105 |  | 1 | $ 105 | | 2 | $ 95 |  | 2 | $ 95 | | 3 | $ 85 |  | 3 | $ 86 | | 4 | $ 75 |  | 4 | $ 78 | | 5 | $ 65 |  | 5 | $ 71 |   Explain your reasoning.  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  4. Write a real-life situation that could be represented by the function f(t) = 3000(1.07)t . Explain.  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| **COLLABORATIVE ACTIVITY** |
| Name of Assessment Task: |
| |  |  |  | | --- | --- | --- | | **Card Set A: (Verbal)** | | | | A1.  The population of Waycross in 2009 is 14,200. The population has dropped by 4% since 2000. | A2.  The purchase price of a new car is $18,000. The value of the car depreciates yearly by 12%. | A3.  A mouse population of 14,200 is increasing in size at a rate of 4% per year. | | A4.  The value of a certain stock is $18,000 and is growing annually at a rate of 12%. | A5.  A mechanic charges $40 per hour plus a flat rate of $25. | A6.  The temperature was 25° and it dropped 40° every 30 minutes. | | A7.  Mary had *n* dollars in the bank and spends *m* dollars per week on CD’s. | A8.  Mom pays Mary *n* dollars each semester to keep her room clean and *m* dollars per A on her report card. | A9.  Mary’s hometown is experiencing a yearly population growth of *n%*. The original population is *m*. | | A10.  The price of oil is *m* per barrel. Because of low demand, the price has decreased n% per week. |  |  | |

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| Card Set B (Modeling Function) | |
| B1.  y = 40x + 25 | B2.  y = -40x + 25 |
| B3.  y = 14,200(.96)t | B4.  y = 14,200(1.04)t |
| B5.  y = 18,000(.88)t | B6.  y = 18,000(1.12)t |
| B7.  y = mx + n | B8.  y = -mx + n |
| B9. | B10. |

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| Card Set C (Possible Graphs) | |
| C1 | C2 |
| C3 | C4 |

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| **Collaborative Activity Instructions:** |
| 1) You have been grouped in pairs.  2) You are given card sets A (verbal situations) and B  (modeling functions), already cut apart.  3) Read each situation carefully. You and your partner should  match each situation from card set A with a modeling  function from card set B. Discuss to ensure that you both  agree.  4) Once you are sure that you have completed the matching  correctly and you are instructed to do so, take out card set C.  Match card set C to card sets A & B previously matched.  Note, more than one set of matched A&B cards will be used  with each graph in set C.  5) On the construction paper provided, glue or tape the  matching sets down (grouped under card set C). Be sure to  explain in writing your reasoning. |

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